$$C_+ \; characteristic \; line \rightarrow \partial(\theta-\nu) = -\frac{1}{\sqrt{M^2-1}+1/\tan\theta}\frac{\partial r}{r} \rightarrow \boxed{\begin{array}{l} \dfrac{\partial(\theta-\nu)}{\partial K_+} = -\left(\dfrac{\sin\mu \cdot \sin\theta}{r}\right) \\ \partial K_+ = \dfrac{\partial r}{\left[\sin(\theta+\mu)\right]} = \dfrac{\partial x}{\cos(\theta+\mu)} \end{array}}$$

$$C_- \; characteristic \; line \rightarrow \partial(\theta+\nu) = \frac{1}{\sqrt{M^2-1}-1/\tan\theta}\frac{\partial r}{r} \rightarrow \boxed{\begin{array}{l} \dfrac{\partial(\theta+\nu)}{\partial K_-} = \left(\dfrac{\sin\mu \cdot \sin\theta}{r}\right) \\ \partial K_- = \dfrac{\partial r}{\left[\sin(\theta-\mu)\right]} = \dfrac{\partial x}{\cos(\theta-\mu)} \end{array}}$$

$$(x,y)_{3D} \rightarrow (x,r)_{axi}$$

$$\boxed{\begin{array}{l} C_+ \; characteristic \; line \rightarrow \dfrac{\partial}{\partial K_+}(\theta-\nu) = -\dfrac{\sin\mu \cdot \sin\theta}{r} \\ C_- \; characteristic \; line \rightarrow \dfrac{\partial}{\partial K_-}(\theta+\nu) = \dfrac{\sin\mu \cdot \sin\theta}{r} \end{array}}$$

$$\boxed{\begin{array}{l} Slope(C_+) = \theta+\mu \\ Slope(C_-) = \theta-\mu \end{array}}$$

1) *Internal Flow Field Solution Process*

$$Assume \; Straight \; \text{Initial} \; Characteristic \; Line \rightarrow \boxed{\begin{array}{l} Slope(C_+)^{(0)} = \theta_2+\mu_2 \\ Slope(C_-)^{(0)} = \theta_1-\mu_1 \end{array}}$$

$$\rightarrow\rightarrow Solve \; for \; \{x_3, r_3\} \rightarrow$$

$$\boxed{\begin{array}{l} r_3 = \dfrac{\tan\left[Slope(C_-)^{(j)}\right]\cdot\tan\left[Slope(C_+)^{(j)}\right](x_1-x_2)+\tan\left[Slope(C_-)^{(j)}\right]\cdot r_2-\tan\left[Slope(C_+)^{(j)}\right]\cdot r_1}{\tan\left[Slope(C_-)^{(j)}\right]-\tan\left[Slope(C_+)^{(j)}\right]} \\ x_3 = \dfrac{(r_2-r_1)+x_1\cdot\tan\left[Slope(C_-)^{(j)}\right]-x_2\cdot\tan\left[Slope(C_+)^{(j)}\right]}{\tan\left[Slope(C_-)^{(j)}\right]-\tan\left[Slope(C_+)^{(j)}\right]} \end{array}}$$

.....*Calculate increments in* K_+, K_-

$$\boxed{\begin{array}{l} \Delta K_+ = \dfrac{x_3-x_2}{\cos\left[Slope(C_+)^{(j)}\right]} \\ \Delta K_- = \dfrac{x_3-x_1}{\cos\left[Slope(C_-)^{(j)}\right]} \end{array}}$$

.....*Advance Characteric Line Invariants*

$$\boxed{\begin{array}{l} C_+ \; characteristic \; line \rightarrow \theta_3-\nu_3 = \theta_2-\nu_2-\dfrac{\sin\mu_2 \cdot \sin\theta_2}{r_2}\Delta K_+ \\ C_- \; characteristic \; line \rightarrow \theta_3+\nu_3 = \theta_1+\nu_1+\dfrac{\sin\mu_1 \cdot \sin\theta_1}{r_1}\Delta K_- \end{array}}$$

.....*Solve for* θ_3

$$\boxed{\begin{array}{l} \theta_3 = \dfrac{\theta_2-\nu_2-\dfrac{\sin\mu_2 \cdot \sin\theta_2}{r_2}\Delta K_+ + \theta_1+\nu_1+\dfrac{\sin\mu_1 \cdot \sin\theta_1}{r_1}\Delta K_-}{2} = \\ \dfrac{(\theta_2+\theta_1)+(\nu_1-\nu_2)+\dfrac{\sin\mu_1 \cdot \sin\theta_1}{r_1}\Delta K_- - \dfrac{\sin\mu_2 \cdot \sin\theta_2}{r_2}\Delta K_+}{2} \end{array}}$$

.....*Solve for* ν_3

$$\boxed{\begin{array}{l} \nu_3 = \dfrac{\theta_1+\nu_1+\dfrac{\sin\mu_1 \cdot \sin\theta_1}{r_1}\Delta K_- - \left(\theta_2-\nu_2-\dfrac{\sin\mu_2 \cdot \sin\theta_2}{r_2}\Delta K_+\right)}{2} = \\ \dfrac{(\theta_1-\theta_2)+(\nu_1+\nu_2)+\dfrac{\sin\mu_1 \cdot \sin\theta_1}{r_1}\Delta K_+ + \dfrac{\sin\mu_2 \cdot \sin\theta_2}{r_2}\Delta K_-}{2} \end{array}}$$

$$\rightarrow\rightarrow\rightarrow .....Solve \; for \; (\theta_3, \nu_3) \rightarrow M_3, \mu_3$$

.....Re*calculate*

$$Slope(C_+)^{(j)} = \frac{\theta_2+\mu_2+\theta_3+\mu_3}{2}$$

$$Slope(C_-)^{(j)} = \frac{\theta_1-\mu_1+\theta_3-\mu_3}{2}$$

Iterate from $\rightarrow\rightarrow$ *to* $\rightarrow\rightarrow\rightarrow$